animal and a powerful Leyden battery, it was concluded that the quantity of force in each shock of the former was very great. It was also ascertained by all the tests capable of bearing on the point, that the current of electricity was, in every case, from the anterior parts of the animal through the water or surrounding conductors to the posterior parts. The author then proceeds to express his hope that by means of these organs and the similar parts of the Torpedo, a relation as to *action* and *re-action* of the electric and nervous powers may be established experimentally; and he briefly describes the form of experiment which seems likely to yield positive results of this kind.

December 20, 1838.

JOHN GEORGE CHILDREN, Esq., V.P., in the Chair.

Prof. Louis Agassiz, and Prof. Carl. Fred. Philip von Martius, were severally elected Foreign Members of the Society.

A paper was read, entitled, "On the Curvature of Surfaces." By John R. Young, Esq. Communicated by John W. Lubbock, Esq., M.A., V.P. and Treas. R.S.

The principal object of this paper is, to remove the obscurity in which that part of the theory of the curvature of surfaces which relates to umbilical points has been left by Monge and Dupin, to whom, however, subsequently to the labours of Euler, we are chiefly indebted for a comprehensive and systematic theory of the curvature of surfaces. In it the author shows, that the lines of curvature at an umbilic are not, as at other points on a surface, two in number, or, as had been stated by Dupin, limited; but that they proceed in every possible direction from the umbilic.

The obscurity complained of is attributed to the inaccurate conceptions entertained by Monge and Dupin, of the import of the symbol $\frac{0}{0}$ in the analytical discussion of this question, the equation which determines the directions of the lines of curvature taking the form

$$0\left(\frac{dy}{dx}\right)^2 + 0\left(\frac{dy}{dx}\right) + 0 = 0$$

at an umbilic. After stating that Dupin has been guided by the determination of the differential calculus, the author remarks, that in no case is the differential calculus competent to decide whether $\frac{0}{0}$, the form which a general analytical result takes in certain particular hypotheses, as to the arbitrary quantities entering that result, has or has not innumerable values. He then states the principle, that those values of the arbitrary quantities (and none else) which render the equations of condition indeterminate must also render the final re-

sult, to which they lead, equally indeterminate; and that, therefore, when such result assumes the form $\frac{0}{0}$, its true character is to be tested by the equations that have led to it, after these have been modified by the hypothesis from which that form has arisen.

In a "Mémoire sur la Courbure des Surfaces," (Journal de l'École Polytechnique, Tom. XIII.), Poisson has arrived at the conclusion, that the number of lines of curvature passing through an umbilical point is infinite, and that those selected by Dupin differ from the others only by satisfying an additional differential equation; those others equally satisfying the conditions of a line of curvature. These are precisely the conclusions arrived at by the author. As, however, he considers that the mode of investigation pursued by Poisson is peculiar and ill adapted to the objects apparently in view, namely, to reconcile the results of Monge and Dupin and to remove their obscurities, he was induced to investigate some of the more important properties of curve surfaces, by a method somewhat different from that usually employed.

Adopting $$Z = F(X, Y)$$
as the general equation of any surface; by attributing to X, Y, Z, increments x, y, z, and assuming that the axis Z coincides with the normal to the surface, or that the plane $x\,y$ is parallel to the tangent plane, an equation equivalent to, and nearly identical with, Dupin's equation of his indicatrix, is readily deduced. From this are immediately derived some properties of the radii of curvature, first shown by Dupin; and likewise the theorem of Meusnier. The author then enters upon the subject of the lines of curvature.

From the equations
$$A = 0, \qquad B = 0,$$
of the normal to the surface at a point on it, the equations of the normal at a point near to the former are determined. That these normals may intersect, which is the condition giving the directions of the lines of curvature, the two sets of equations must simultaneously exist; and hence are deduced the differential equations of condition for the lines of curvature,

$$\frac{dA}{dx} + \frac{dA}{dy} \cdot \frac{dy}{dx} = 0, \qquad \frac{dB}{dx} + \frac{dB}{dy} \cdot \frac{dy}{dx} = 0.$$

By this method, which fundamentally is not very different from that of Monge, substituting the usual expressions for A and B, the equation that determines the directions of the lines of curvature is deduced, in the form in which it had been previously given by Monge and Dupin.

This final equation becoming at an umbilic of the form,

$$0\left(\frac{dy}{dx}\right)^2 + 0\left(\frac{dy}{dx}\right) + 0 = 0,$$

in which $\frac{dy}{dx}$ *may* be indeterminate, the author inquires how this in-

determinate form will affect the equations of condition. As by this supposition, these are reduced to equations from which would result the conditions that would render all the coefficients of the determining equation 0, it is inferred that $\frac{d\,y}{d\,x}$ *must* be indeterminate, and that therefore, at an umbilic there issue lines of curvature in all directions.

Of these lines of curvature, it is possible that some may be distinguished from others, by proceeding from the point in more intimate contact with the osculating sphere, and it is therefore necessary to determine the analytical character of such particular lines of curvature. With this view, the author resumes the equation of the normal in the immediate vicinity of the umbilic. He then points out, that a straight line, whose equations contain the second differential coefficients, thus involving a new condition, will coincide more nearly with this normal, than can any straight line not having that condition. That the lines may intersect in the centre of the osculating sphere, their equations must simultaneously exist; and thus, that which most nearly coincides with the normal in the immediate vicinity of the umbilic has the new conditions,

$$\frac{d^2\,\mathrm{A}}{d\,x^2} + 2\,\frac{d^2\,\mathrm{A}}{d\,x\,d\,y}\cdot\frac{d\,y}{d\,x} + \frac{d^2\,\mathrm{A}}{d\,y^2}\cdot\frac{d\,y^2}{d\,x^2} = 0,$$

$$\frac{d^2\,\mathrm{B}}{d\,x^2} + 2\,\frac{d^2\,\mathrm{B}}{d\,x\,d\,y}\cdot\frac{d\,y}{d\,x} + \frac{d^2\,\mathrm{B}}{d\,y^2}\cdot\frac{d\,y^2}{d\,x^2} = 0,$$

in addition to the former ones.

From this it appears, that when the direction of a line of curvature issuing from an umbilic is such as to fulfil, besides the ordinary conditions, the foregoing new conditions, that line of curvature will lie more closely to the osculating sphere than any other not satisfying these additional equations. These new conditions arise from differentiating the preceding ones with respect to x and y, considered as dependent, regarding $\frac{d\,y}{d\,x}$ as constant; and as these are equivalent to a single condition (Monge's and Dupin's equation) it will be sufficient to differentiate this, under the above restrictions, in order to obtain a single condition equivalent to the new ones. As this single condition will appear under the form of an equation of the third degree in $\frac{d\,y}{d\,x}$, there will, in general, be at least one line of curvature, proceeding from the umbilic, of more than ordinary closeness to the osculating sphere; and there may be three. If, indeed, this equation of the third degree should, like that of the second from which it is deduced, be identical for the coordinates of the umbilic, it is obvious from the investigation, that we must then proceed to another differentiation; and so on, till we arrive at a determinate equation, the real roots of which will make known the number and directions of the lines of closest contact.

When, however, the author remarks in conclusion, all the lines of curvature issuing from the umbilic are equally close to the osculating sphere, then these successive differentiations will either at length exhaust the coefficients, and thus no determinate equation will arise ; or else they will conduct to an equation whose roots are all imaginary : and one or other of these circumstances must always take place at the vertex of a surface of revolution.

The Society adjourned over the Christmas Recess to meet again on the 10th January next.

January 10, 1839.

JOHN WILLIAM LUBBOCK, Esq., V.P. and Treas., in the Chair.

William James Frodsham, and John Hilton, Esquires, were severally elected Fellows of the Society.

A paper was read, entitled, " On the Laws of Mortality." By Charles Jellicoe, Esq. Communicated by P. M. Roget, M.D., Sec. R.S.

The author, considering that the variations and discrepancies in the annual decrements of life which are exhibited in the tables of mortality hitherto published would probably disappear, and that these decrements would follow a perfectly regular and uniform law, if the observations on which they are founded were sufficiently numerous, endeavours to arrive at an approximation to such a law, by proper interpolations in the series of the numbers of persons living at every tenth year of human life. The method he proposes, for the attainment of this object, is that of taking, by proper formulæ, the successive orders of differences, until the last order either disappears, or may be assumed equal to zero. With the aid of such differences, of which, by applying these formulæ, he gives the calculation, he constructs tables of the annual decrements founded principally on the results of the experience of the Equitable Assurance Society.

January 17, 1839.

JOHN FORBES ROYLE, M.D., V.P., in the Chair.

Beriah Botfield, and Peter Hardy, Esquires, were severally elected Fellows of the Society.

A paper was read, entitled, " On the state of the Interior of the Earth." By W. Hopkins, Esq. A.M., F.R.S., Second Memoir. " On the Phenomena of Precession and Nutation, assuming the Fluidity of the Interior of the Earth."